AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST TARGET - JEE - TEST - 14
SOLUTION Date 11-08-2019

[PHYSICS]

1. Kepler's second law is a consequence of conservation of angular momentum

2. According to Kepler's first law, every planet moves in an elliptical orbit with the sun situated at one of the foci of the ellipse.

   In options (a) and (b) sun is not at a focus while in (c) the planet is not in orbit around the sun. Only (d) represents the possible orbit for a planet.

3. Kepler's law $T^2 \propto R^3$

4. During path DAB planet is nearer to sun as comparison with path BCD. So time taken in travelling DAB is less than that for BCD because velocity of planet will be more in region DAB.

5. Time period of a revolution of a planet,

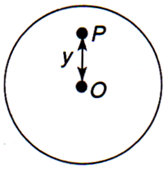
$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM_S}{r}}} = \frac{2\pi r^{3/2}}{\sqrt{GM_S}}$$

6. Gravitational force is independent of the medium. Thus, gravitational force will be same i.e., F.

7. In arrangement 1, both forces act in the same direction. In arrangement 3, both the forces act in opposite direction. This alone decides in favour of option (a),

8. If a point mass is placed inside a uniform spherical shell, the gravitational force on the point mass is zero. Hence, the gravitational force exerted by the shell on the point mass is zero.

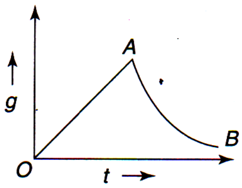
9. $g_d = g\left(1 - \frac{d}{g}\right)$



or $g_d = g\frac{R-d}{R}$

or $g_d = \frac{gy}{R}$ or $g_d \propto y$

So, within the Earth, the acceleration due to gravity varies linearly, with the distance from the centre of the Earth. This explains the linear portion OA of the graphs.



10. The value of g at the height h from the surface of earth

$$g' = g\left(1 - \frac{2h}{R}\right)$$

The value of g at depth x below the surface of earth

$$g' = g\left(1 - \frac{x}{R}\right)$$

These two are given equal, hence $\left(1 - \frac{2h}{R}\right) = \left(1 - \frac{x}{R}\right)$

On solving, we get $x = 2h$

11. Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR$ $\therefore$ $g \propto \rho R$

or $\frac{g_m}{g_e} = \frac{\rho_m}{\rho_e} \cdot \frac{R_m}{R_e}$

$$\left[\text{As } \frac{g_m}{g_e} = \frac{1}{6} \text{ and } \frac{\rho_e}{\rho_m} = \frac{5}{3} \text{(given)}\right]$$

$$\therefore \quad \frac{R_m}{R_e} = \left(\frac{g_m}{g_e}\right)\left(\frac{\rho_e}{\rho_m}\right) = \frac{1}{6} \times \frac{5}{3} \therefore R_m = \frac{5}{18}R_e$$

12. Acceleration due to gravity $g = \frac{GM}{R^2}$

$$\therefore \quad \frac{g_{\text{moon}}}{g_{\text{earth}}} = \frac{M_{\text{moon}}}{M_{\text{earth}}} \cdot \frac{R^2_{\text{earth}}}{R^2_{\text{moon}}} = \left(\frac{1}{80}\right)\left(\frac{4}{1}\right)^2$$

$$g_{\text{moon}} = g_{\text{earth}} \times \frac{16}{80} = \frac{g}{5}$$

13. Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR$

$$\therefore \quad g_1 : g_2 = R_1\rho_1 : R_2\rho_2$$

14. $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{4} = g\left(1 - \frac{d}{R}\right) \Rightarrow d = \frac{3R}{4}$

15. We know $g = \frac{GM}{R^2} = \frac{GM}{(D/2)^2} = \frac{4GM}{D^2}$

If mass of the planet = M_0 and diameter of the planet = D_0. Then $g = \frac{4GM_0}{D_0^2}$.

16. $\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{R}{R+2R}\right)^2 = \frac{1}{9} \quad \therefore g' = \frac{g}{9}$.

17. Acceleration due to gravity on earth is

$$g = \frac{GM_E}{R_E^2} \qquad \text{(i)}$$

As $\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} \Rightarrow M_E = \rho \frac{4}{3}\pi R_E^3$

Substituting this value in Eq. (i), we get

$$g = \frac{G\left(\rho \frac{4}{3}\pi R_E^3\right)}{R_E^2} = \frac{4}{3}\pi\rho G R_E \text{ or } \rho = \frac{3g}{4\pi G R_E}$$

18. $g = \frac{GM}{R^2}$

$$\frac{\Delta g}{g} \times 100 = 2\frac{\Delta R}{R} \times 100 = 2 \times 1\% = 2\%$$

19. Gravitational P.E. = $m \times$ gravitational potential

$U = mV$

So the graph of U will be same as that of V for a spherical shell.

20. $\Delta U = U_2 - U_1 = \frac{mgh}{1+\frac{h}{R_e}} = \frac{mgR_e}{1+\frac{R_e}{R_e}} = \frac{mgR_e}{2}$

$$\Rightarrow \quad U_2 - (-mgR_e) = \frac{mgR_e}{2} \Rightarrow U_2 = -\frac{1}{2}mgR_e$$

21. $\Delta K.E. = \Delta U$

$$\frac{1}{2}MV^2 = GM_eM\left(\frac{1}{R} - \frac{1}{R+h}\right) \qquad \text{(i)}$$

Also $g = \frac{GM_e}{R^2}$ (ii)

On solving (i) and (ii) $h = \frac{R}{\left(\frac{2gR}{V^2} - 1\right)}$

22. Potential energy $U = \frac{-GMm}{r} = -\frac{GMm}{R+h}$

$$U_{\text{initial}} = -\frac{GMm}{3R} \text{ and } U_{\text{final}} = -\frac{-GMm}{2R}$$

Loss in PE = gain in $KE = \frac{GMm}{2R} - \frac{GMm}{3R} = \frac{GMm}{6R}$

23. $v = -\frac{GMM}{R} = -\frac{GM^2}{R}$

24. Before collision, $PE = mV = -\frac{GMm}{r}$

After collision, velocity will be zero. The wreckage will come to rest. The energy will be only potential energy.

$$PE = -\frac{GMm}{r} = -\frac{2GMm}{r} \text{ Ratio} = 1/2$$

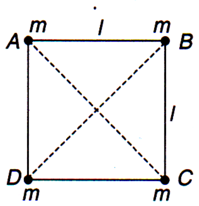
25. $\Delta u = -\left(\frac{GMm}{R+h}\right) - \left(-\frac{GMm}{R}\right)$

$H = 3R$

$\Delta u = -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3GMm}{4R^2} \times R$

$\Delta u = \frac{3}{4} mgR$

26.



From figure

$$AB = BC = CD = AD = l$$

27. $U = \frac{-GMm}{r}, K = \frac{GMm}{2r} \text{ and } E = \frac{-GMm}{2r}$

For a satellite U, K and E vary with r and also U and E remain negative whereas K remains always positive.

28. $v = \sqrt{\frac{GM}{r}}$ if $r_1 > r_2$ then $v_1 < v_2$

Orbital speed of satellite does not depend upon the mass of the satellite.

29. $v=\sqrt{\frac{GM}{R+h}}$

For first satellite $h=0,\ v_1=\sqrt{\frac{GM}{R}}$

For second satellite $h=\frac{R}{2},\ v_2=\sqrt{\frac{2GM}{3R}}$

$$v_2=\sqrt{\frac{2}{3}}v_1=\sqrt{\frac{2}{3}}v$$

30. Since the planet is at the centre, the focus and centre of the elliptical path coincide and the elliptical path becomes circular and the major axis is nothing but the diameter. For a circular path:

$$\frac{mv^2}{r}=\sqrt{\frac{GM}{r^2}}m$$

Also $T=\frac{2\pi r}{v}=\frac{2\pi r^{3/2}}{\sqrt{GM}}\Rightarrow T^2=\frac{4\pi^2 r^3}{GM}$

$\Rightarrow\quad r=\left(\frac{GMT^2}{4\pi^2}\right)^{1/3}=\text{Radius}$

$\Rightarrow\quad$ Diameter (major axis) $=2\left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$

[CHEMISTRY]

31.

ΔE and ΔH both are zero in case of cyclic process. [Also, for isothermal free or reversible expansion of ideal gas, ΔE and ΔH both are zero].

32.

During adiabatic process, no heat is exchanged with surrounding. Hence, $q=0$.

From $\Delta E=q+W$ (Work done on the system)

$\Delta E=W$ (Since, $q=0$)

33.

34.

In case of thermodynamic equilibrium $\Delta V, \Delta P, \Delta T$ and Δn all have to be zero.

35.

36.

$\Delta n_g=2\,(\text{of } XY_3)-[1(\text{of } X_2)+3\,(\text{of } Y_2)]=-2$

$\Delta H-\Delta E=\Delta n_g RT$

But, given value is z.

So, $z=\Delta n_g RT$

$$\frac{z}{R}=\Delta n_g T=-2\times(27+273)=-600=-\mathbf{6\times10^2}$$

37.

$$\Delta U = \Delta H - \Delta n_g RT = 41 - 1 \times \frac{8.3}{1000} \times 373 = 41 - 3.0959 = \mathbf{37.9041\ kJ\ mol^{-1}}$$

38.

For (i) $\Delta H = \Delta U$, because $\Delta n_g = 0$

For (ii) $\Delta H < \Delta U$, because Δn_g is negative (–2).

For (iii) $\Delta H > \Delta U$, because Δn_g is positive (+0.5).

39.

For $N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$; $\Delta n_g = 2 - 4 = -2$

For $N_2(g) + 3H_2(g) \longrightarrow 2NH_3(l)$; $\Delta n_g = 0 - 4 = -4$

In both the cases, $\Delta H = \Delta U + \Delta n_g RT$, will give $\boldsymbol{\Delta H < \Delta U}$.

40.

More negative the enthalpy of formation, more is the stability.

41.

H_2, O_2 and H_2O all are in their standard states and 1 mol of water is being prepared.

42.

Subtract the 2nd equation from 1st

$S_r \longrightarrow S_m$; $\Delta H = +2.5$ kJ [Endothermic]

43.

ΔH for $P \longrightarrow 2Q$ is obtained using Hess's law, by adding Eqn. (i), Eqn. (ii) and $2\times$ Eqn. (iii), $\boldsymbol{\Delta H = x + y + 2z}$.

44.

$$W_{\text{expansion}} = -P\Delta V$$
$$= -(1\times 10^5\, Nm^{-2})[(1\times 10^{-2} - 1\times 10^{-3})m^3]$$
$$= -10^5 \times (10\times 10^{-3} - 1\times 10^{-3})\, Nm$$
$$= -10^5 \times 9\times 10^{-3}\, J = -9\times 10^2\, J = \mathbf{-900\ J}$$

45.

$q = 300$ calorie

$W = -P\,\Delta V = -1\times 10$ litre-atm $= -10\times 24.2$ cal $= -242$ cal

$\Delta E = q + W = 300 - 242 = \mathbf{58\ cal}$

46.

$q = \mathbf{+200\ J}$

$W = -P\Delta V = -1\times(20-10) = -10$ atm L

$= -10\times 101.3$ J $= \mathbf{-1013\ J}$

$\Delta E = q + W = (200 - 1013)$ J $= \mathbf{-813\ J}$

47

48.

ΔH for isothermal reversible expansion is **zero.**

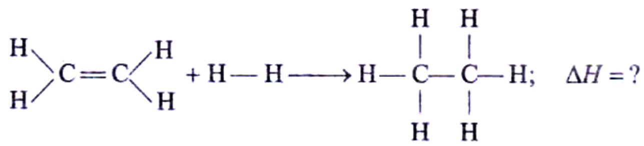
49.

$$W = -2.303 nRT \log \frac{V_2}{V_1}$$
$$= -2.303\times 2\times 8.314\times 300\times \log\frac{50}{5}\ \text{joule}.$$
$$= -11488.285\, J \simeq \mathbf{-11.5\ kJ}$$

50.

As the system starts from A and reaches to A again, whatever the stages may be net energy change is **zero**.

51.



ΔH = Sum of bond energies of reactants – Sum of bond energies of products

$$= 4\,B.E._{C-H} + B.E._{C=C} + B.E._{H-H} - 6\,B.E._{C-H} - B.E._{C-C}$$

$$= 4 \times 410 + 600 + 400 - 6 \times 410 - 350 = 2640 - 2810 = \mathbf{-170\ kJ\ mol^{-1}}$$

52.

$$CH \equiv CH + H_2 \longrightarrow CH_2 = CH_2; \quad \Delta H = ?$$

$$\Delta H = \Delta H_f(CH_2 = CH_2) - \Delta H_f(CH \equiv CH) - \Delta H_f(H_2)$$

$$= y - x - 0 = \mathbf{(y - x)\ J\ mol^{-1}}$$

53.

$$\frac{V_2}{V_1} = \frac{1}{10}$$

$$W \text{ (on the system)} = -2.303nRT \log \frac{V_2}{V_1} = -2.303 \times 1 \times 2 \times 500 \log \frac{1}{10} \text{ cal}$$

$$= +\frac{2.303 \times 2 \times 500}{1000} \text{ kcal} = \mathbf{+2.303\ kcal}$$

55.

56.

It is a combustion reaction. So ΔH is negative. The number of moles of gaseous substances is decreasing (from 7.5 to 6), ΔS is also negative. The reaction occurs when $\boldsymbol{\Delta H > T\Delta S}$.

57.

$$\Delta H = \Delta E + \Delta n_g RT$$

For $\quad H_2O(l) \xrightarrow{373\text{ K}} H_2O(g)$

$$\Delta H = 37.5 + 1 \times 8.3 \times 373 = 37.5 + 3095.9$$

$$= 3133.4\text{ J} = \mathbf{3.1334\ kJ}$$

58.

$$\Delta G = -nFE = -2.303RT \log K$$

$$\log K = \frac{nFE}{2.303RT} = \frac{nE}{0.059} = \frac{2 \times 0.295}{0.059} = 10 = \log 10^{10}$$

$$K = \mathbf{10^{10}}$$

59. Given, $H_2 + O_2 \longrightarrow H_2O_2$; $\Delta H = -188$ kJ ...(1)

$H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$; $\Delta H = -286$ kJ ...(2)

To get,

$$2H_2O_2(\ell) \longrightarrow 2H_2O(\ell) + O_2(g); \ \Delta H = ?$$

Multiply both (1) & (2) by 2 and then subtract (1) from (2) we get

$$\Delta H = -286 \times 2 - (-2 \times 188) = -196 \text{ kJ mol}^{-1}$$

60. Bond energy is the average energy hence the bond energy of $C-H$ bond is

$$= \frac{\text{Dissociation energy of } CH_4}{\text{No. of bond broken}} = \frac{X_1}{4}$$

[MATHEMATICS]

61 Given $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$

operate $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$

$\Rightarrow \begin{vmatrix} 1+a & 1 & 1 \\ -a & b & 0 \\ 0 & -b & c \end{vmatrix} = 0$

$\Rightarrow$ (1 + a) bc – 1(–ac) + 1 (ab) = 0

$\Rightarrow$ bc + ab + ac + abc = 0

$\Rightarrow$ $a^{-1} + b^{-1} + c^{-1} = -1$

62. Here, $\begin{vmatrix} s+c & a & b \\ c & s+a & b \\ c & a & s+b \end{vmatrix}$, Operate $R_1 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$= \begin{vmatrix} s+c & a & b \\ -s & s & 0 \\ -s & 0 & s \end{vmatrix}$, operate $C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix} s+a+b+c & a & b \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix}$

$= (s + a + b + c)s^2 = (s + s)s^2 = 2s^3$.

63. We are given that a > 0

$4b^2 - 4ac < 0 \Leftrightarrow ac - b^2 > 0$

Hence, $ax^2 + 2bx + c > 0$ for all $x \in$ **R**.

Operating $R_3 \to R_3 - xR_1 - R_2$, we find that the value of given determinant $= -(ax^2 + 2bx + c)(ac - b^2) < 0$ for all $\in$ **R**.

64. Operating $R_1 \to R_1 + R_3$, we get

$\begin{vmatrix} 0 & 0 & 2\cos\theta \\ -\sin\theta & \cos\theta & \sin\theta \\ -\cos\theta & -\sin\theta & \cos\theta \end{vmatrix} = 0$

$\Rightarrow$ $2\cos\theta(\sin^2\theta + \cos^2\theta) = 0$

$\Rightarrow$ $\cos\theta = 0 \Rightarrow \cos\theta = \cos\frac{\pi}{2}$

$\Rightarrow$ $\theta = 2n\pi \pm \frac{\pi}{2}, n \in I$

65. Operating

$C_1 \to C_1 + C_2 - C_3$, we get

$$\begin{vmatrix} {}^{10}C_4 + {}^{10}C_5 - {}^{11}C_m & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 + {}^{11}C_7 - {}^{12}C_{m+2} & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 + {}^{12}C_9 - {}^{13}C_{m+4} & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} {}^{11}C_5 - {}^{11}C_m & {}^{10}C_5 & {}^{11}C_m \\ {}^{12}C_7 - {}^{12}C_{m+2} & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{13}C_9 - {}^{13}C_{m+4} & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$$

We note that $m = 5$ makes $C_1 = O$.

66. Here

$$\Delta = \begin{vmatrix} \log x & \log y & \log z \\ \log 2 + \log x & \log 2 + \log y & \log 2 + \log z \\ \log 3 + \log x & \log 3 + \log y & \log 3 + \log z \end{vmatrix}$$

Operate $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} \log x & \log y & \log z \\ \log 2 & \log 2 & \log 2 \\ \log 3 & \log 3 & \log 3 \end{vmatrix}$$

$$= (\log 2)(\log 3) \begin{vmatrix} \log x & \log y & \log z \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 0$

67. **Now** $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$

$$= -1(1 - \cos^2 A) - \cos C(-\cos C - \cos A \cos B) + \cos B(\cos A \cos C + \cos B)$$

$$= -1 + \cos^2 A + \cos^2 C + 2 \cos A \cos B \cos C + \cos^2 B$$

$$= \cos^2 A + \cos^2 B + \cos^2 C - \{1 - 2 \cos A \cos B \cos C\}$$

$= 0.$ **(From trigonometry)**

68. Given

$$\Delta(x)=\begin{vmatrix} x & x^3 & (x-1)^2 \\ 2x & 1 & 3x^2 \\ 2 & 0 & 6x \end{vmatrix},$$

expand with the help of R_1

$= x\{6x-0\} - x^3\{12x^2-6x^2\}$

$+ (x-1)^2\{0-2\}$

$= 6x^2 - 6x^5 - 2(x^2-2x+1)$

$= 6x^2 - 6x^5 - 2x^2 + 4x - 2$

Hence, coefficient of x in $\Delta(x)$ is 4.

69. Here, $A^2 = AA = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\Rightarrow$ $A^n = O$ for all $n \geq 2$

and $f(A) = I + A + A^2 + ... + A^{16} = I + A$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$

70. For a unique solution, $\begin{vmatrix} 1 & 1 & 1 \\ 3 & 6 & 1K \\ \alpha & 2 & 3 \end{vmatrix} K \neq 0,$

i.e., $\alpha \neq \frac{13}{5}$

71. The given system of equations have a nontrivial solution only if

$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2=0, \\ 2 & 3 & -4 \end{vmatrix}$

i.e., (–4k + 6) – k(–12 + 4) + 3 (9 – 2k) = 0,

i.e., if – 4k +6 + 8k + 27 – 6k = 0,

i.e., if – 2k = – 33, i.e., if $k = \frac{33}{2}$

72. Second equation is 5x + 2y – 3z = 2

$\Rightarrow$ 3 (5x + 2y – 3z) = 6

$\Rightarrow$ 15x + 6y – 9z = 6,

which does not agree with the third equation. Hence, the given system is inconsistent, i.e., it has no solution.

73. For no solution $\begin{vmatrix} \alpha & 1 & 1 \\ 1 & ga & 1 \\ 1 & 1 & \theta \end{vmatrix} = 0,$

operate $C_2 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} \alpha+2 & 1 & 1 \\ \alpha+2 & \alpha & 1 \\ \alpha+2 & 1 & \alpha \end{vmatrix} = 0$

$\Rightarrow \quad (\alpha+2)\begin{vmatrix}1 & 1 & 1\\ 1 & \alpha & 1\\ 1 & 1 & \alpha\end{vmatrix} = 0$

operate $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$

$\Rightarrow \quad (\alpha+2)\begin{vmatrix}1 & 0 & 0\\ 1 & \alpha-1 & 0\\ 1 & 0 & \alpha-1\end{vmatrix} = 0$

$\Rightarrow \quad (\alpha + 2)(\alpha - 1)^2 = 0$

$\Rightarrow \quad \alpha = -2$ or $\alpha = 1$

But when $\alpha = 1$, then the system becomes homogeneous which always admits of a solution (the trivial solution). So $\neq 1$

74. $\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 2\\ 1 & 2 & \lambda\end{vmatrix} = 0 \Rightarrow \begin{vmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 0 & 1 & \lambda-1\end{vmatrix} = 0$

$\Rightarrow \quad \lambda - 1 - 1 = 0 \Rightarrow \lambda = 2$

75. $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$, reduces the determinant to $\begin{vmatrix}x^2-1 & x-1 & 0\\ 2x-2 & x-1 & 0\\ 3 & 3 & 1\end{vmatrix} = (x^2-1)(x-1) - 2(x-1)^2$

$= (x - 1)^2 (x + 1 - 2)$

$= (x - 1)^3$

76. $u_n = an^2 + bn + c - a(n - 1)^2 - b(n - 1) - c$

$= 2an + b$

$\therefore \quad u_1 = 2a + b, u_2 = 4a + b, u_3 = 6a + b$

Clearly, u_1, u_2, u_3 are in A.P. having c.d.2a. Now,

$\Delta = \begin{vmatrix}u_1 & 2a & 2a\\ 1 & 0 & 0\\ 7 & 1 & 1\end{vmatrix} = 0$. $\quad \begin{bmatrix}C_3 \to C_3 - C_2\\ C_2 \to C_2 - C_1\end{bmatrix}$

77. (A) $\Delta(=0) = \Delta'(0) = 0$

$\therefore \quad \Delta(k)$ is divisible by k^2.

78. $A(\text{adj.}(A)) = |A|\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \quad \lambda = |A|$

Now, $|A| = \begin{vmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{vmatrix}$

$= \cos^2\theta + \sin^2\theta = 1$

79. If 'A' is an orthogonal matrix, then

$AA^T = 1$

Here,

$AA^T = \frac{1}{9}\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ x & 2 & y\end{bmatrix}\begin{bmatrix}1 & 2 & x\\ 2 & 1 & 2\\ 2 & -2 & y\end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix}9 & 0 & x+4+2y\\ 0 & 9 & 2x+2-2y\\ x+4+2y & 2x+2-2y & x^2+4+y^2\end{bmatrix}$

$\Rightarrow \quad x + 2y + 4 = 0, \qquad 2x + 2 - 2y = 0$

and $x^2 + y^2 + 4 = 9$

$\Rightarrow$ x = – 2, y = – 1

$\Rightarrow$ x + y = – 3

80. By property, adj (adj A) = $|A|^{n-2}$ A

81. By property, adj A^T – $(\text{adj } A)^T$ = O* null matrix

82. det (AA^{-1}) = det (I)

$\Rightarrow$ det (A) det(A^{-1}) = 1

83. As $\Delta z \neq 0$, for no solution $\Delta = 0$

$$\Rightarrow \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -1 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda = 3.$$

Hence (A) is the correct answer.

84. $$\Delta = \begin{vmatrix} \sin px & p\cos px & -p^2 \sin px \\ -p^3 \cos px & p^4 \sin px & p^5 \cos px \\ -p^6 \sin px & -p^7 \cos px & p^8 \sin px \end{vmatrix}$$

Taking $-p^6$ common from R_3 , R_1 and R_3 becomes identical

Hence (A) is the correct answer.

85. The determinant = $\begin{vmatrix} \alpha^2 & 0 & 1 \\ 0 & \beta^2 & 1 \\ -\gamma^2 & -\gamma^2 & 1+\gamma^2 \end{vmatrix} = \alpha^2\beta^2\gamma^2 + \alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2$

$$\alpha^2\beta^2\gamma^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma) = 1 + 1 - 2 \times 1 \times 1 = 0.$$

Hence (B) is the correct answer.

86. By putting x = 0 on both sides of the equation we have $h = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 9$

Differentiating both sides and then putting x = 0, we get g = -5

Hence (D) is the correct answer.

87. $[\sin^2\theta] = 0 \quad \sin^2\theta \neq 1$

$= 1, \sin^2\theta = 1$

if $\sin^2\theta \neq 1 \Rightarrow D = 2\sin\theta\cos\theta - 2i - 1$

$\text{Re}(D) = 2\sin\theta\cos\theta - 1$

$-2 \leq \text{Re}(D) \leq 0$

$-\frac{3\pi}{4} \leq \arg D \leq -\frac{\pi}{2}$

If $\sin^2\theta = 1$, $\sin\theta = \pm 1$, $\cos\theta = 0$

$\text{Arg}(D) = \arg(1 - 2i)$ or $\arg(-1 - 2i)$

Hence (D) is the correct answer.

88.

$$f(x) = x^2(x-1)\begin{vmatrix} 1 & x & x+1 \\ 2 & (x-1) & x+1 \\ 3 & (x-2) & x+1 \end{vmatrix}$$

Taking common x from R_2 and $x(x-1)$ from R_3

$$= x^2(x^2-1)\begin{vmatrix} 1 & x & 1 \\ 2 & (x-1) & 1 \\ 3 & (x-2) & 1 \end{vmatrix} = x^2(x^2-1)\begin{vmatrix} 1 & x & 1 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{vmatrix}$$

Operating $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$= x^2(x^2 - 1) \times 0 = 0.$

So $f(100) = 0$.

Hence (A) is the correct answer.

89. Since $\Delta = \overline{\Delta}$ $\therefore$ Δ is real only

Hence (A) is the correct answer

90.

We can write the given equations as

AX = B ...(1)

Where, $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$

Since, $|A| = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3(-3+2) - 1(2+1) + 2(4+3) = -3 - 3 + 14 = 8 \neq 0$

From (1), we have $X = A^{-1}B$ ...(2)

Now,

$A_{11} = -1,\quad A_{12} = -3,\quad A_{13} = 7$
$A_{21} = 3,\quad A_{22} = 1,\quad A_{23} = -5$
$A_{31} = 5,\quad A_{32} = 7\quad A_{33} = -11$

Let, $C = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -1 & -3 & 7 \\ 3 & 1 & -5 \\ 5 & 7 & -11 \end{bmatrix}$

$\text{adj } A = \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$

$A^{-1} = \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$

$A^{-1}B = \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -3 & -9 & +20 \\ -9 & -3 & +28 \\ 21 & +15 & -44 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

Hence, from (2) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \Rightarrow x = 1,\ y = 2,\ z = -1.$

Hence (A) is correct.